The Double Adaptivity Paradigm or How to circumvent discrete inf-sup conditions of Babuska and Brezzi?

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In the mixed formulation of the ideal Petrov-Galerkin method with optimal test functions, one solves for an approximate solution coming from a discrete trial space, along with the Riesz representation of the corresponding residual coming from the exact test space. The residual provides an ideal a-posteriori error estimate providing a basis for adaptive refinements of trial space. This is the outer adaptivity loop.

To arrive at a practical method, we need to approximate somehow the residual. In the standard DPG method this is done by employing a sufficiently large discrete (enriched) test subspace of the test space. For (benign) single scale problems, this can be done by implementing elements of higher (enriched) order and constructing appropriate Fortin operators to assess the lost of stability due to the approximation of the exact residual. The situation is quite different for singular perturbation problems where one strives for the robustness, i.e. uniform stability with respect to the perturbation parameter.

Alternatively, with the given approximate trial space, one can solve for the approximate residual ADAPTIVELY. This is the inner adaptivity loop. For singular perturbation problems the challenge comes from the need for a ROBUST a-posteriori error estimation technique.

We propose an inner adaptivity loop built upon the classical duality theory and a-posteriori error estimation based on duality gap estimate (the classical hypercircle methodology). The methodology will be illustrated with a convection-dominated diffusion ("confusion") problem.

The double adaptivity algorithm delivers solutions for the diffusion constant $\epsilon = 10^{-7}$ in a fully automatic mode. The adapted trial meshes with the corresponding adaptively obtained test meshes do NOT satisfy the robust inf-sup condition.